The 3DVAR Filter for Dissipative Systems

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Orientation

- **[BLLMSS11]**: C. Brett, A. Lam, K.J.H. Law, D. McCormick, M. Scott and A.M. Stuart. "Accuracy and Stability of Filters for Dissipative PDEs". PhysicaD **245**(2013), 34–45. `http://arxiv.org/abs/1110.2527`
- **[BLSZ12]** D. Blömker, K.J.H. Law, A.M. Stuart and K.Zygalakis. "Accuracy and Stability of The Continuous-Time 3DVAR Filter for The Navier-Stokes Equation". `http://arxiv.org/abs/1210.1594`
- **[BSS13]** K.J.H. Law, A. Shukla and A.M. Stuart. "Analysis of the 3DVAR Filter for the Partially Observed Lorenz '63 Model". `http://arxiv.org/abs/1212.4923`

Unstable dynamical systems can be stabilized, and hence the solution recovered from noisy data, provided:

- Observe enough of the system: the unstable modes.
- Weight the observed data sufficiently over the model.

Outline

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Dissipative Quadratic Dynamical Systems

A and $B(\cdot,\cdot)$ densely defined linear and bilinear forms on $\big(H, \langle\cdot,\cdot\rangle, |\cdot|\big)$. $f \in H$. $\big(V, \|\cdot\|\big)$ compact in H.

$$\frac{dv}{dt} + Av + B(v,v) = f, \quad v(0) = u$$

Here

$$\begin{aligned} \exists \lambda > 0: \quad & \langle Aw, w\rangle \geq \lambda \|w\|^2, \quad \forall w \in V; \\ & \langle B(w,w), w\rangle = 0, \quad \forall w \in V. \end{aligned}$$

Introduce semigroup notation:

$$v_j = v(jh), \qquad v_{j+1} = \Psi(v_j).$$

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Filtering Problem

Projections:

$$\mathrm{P} : H \to H, \quad \mathrm{Q} = I - \mathrm{P}.$$

Observations:

$$v_{j+1} = \Psi(v_j).$$

$$y_j = \mathrm{P}v_j + \xi_j, \quad \xi_j \sim N(0, \Gamma), \; i.i.d.$$

$$Y_j = \{y_i\}_{i=1}^{j}.$$

Filtering Distribution: find $\mathbb{P}(v_j | Y_j)$.

Do we observe enough to accurately and stably recover the signal? Interaction between P and the dynamics is key.

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Lorenz '63

$$\begin{aligned}\dot{v}_1 &= \alpha(v_2 - v_1),\\ \dot{v}_2 &= -\alpha v_1 - v_2 - v_1 v_3,\\ \dot{v}_3 &= v_1 v_2 - b v_3 - b(r+\alpha), \quad v_k(0) = u_k\end{aligned}$$

We will be interested in the choice:

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Lorenz '96

$$\dot{v}_k = v_{k-1}\Big(v_{k+1} - v_{k-2}\Big) - v_k + f, \quad k = 1, \cdots, K$$
$$v_0 = v_K,\ v_{-1} = v_{K-1},\ v_{K+1} = v_1, \quad v_k(0) = u_k.$$

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Navier-Stokes on a 2D Torus

2D Navier-Stokes as ODE on H :

$$H = \left\{ u \in L^2(\mathbb{T}^2) \middle| \nabla \cdot u = 0, \int_{\mathbb{T}^2} u\,dx = 0 \right\}, \ \mathrm{norm}\,|\cdot|.$$

$$\frac{dv}{dt} + \nu A v + F(v) = f, \quad v(0) = u.$$

Let $A\varphi_k = \lambda_k \varphi_k$ and define

$$\mathrm{P} : H \mapsto \left\{ \varphi_k(x), |k|^2 \le \frac{\lambda}{4\pi^2} \right\},$$

$$\mathrm{Q} : H \mapsto \left\{ \varphi_k(x), |k|^2 > \frac{\lambda}{4\pi^2} \right\}.$$

Outline

3DVAR: Approximate Gaussian Filter

- Impose the (3DVAR) Gaussian approximation:

$$\mathbb{P}(v_j|Y_j) \approx N(\widehat{m}_j, \widehat{C}) \mapsto \mathbb{P}(v_{j+1}|Y_j) \approx N(\Psi(\widehat{m}_j), C)$$
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- Kalman Mean Update:

$$\widehat{m}_{j+1} = (I - KP)\Psi(\widehat{m}_j) + Ky_{j+1}.$$

- Kalman Covariance Update:

$$K = CP^*(PCP^* + \Gamma)^{-1}, \quad \widehat{C} = (I - KP)C.$$

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Intuition for Stabilization via Data

$$v_{j+1} = \Psi(v_j)$$
$$v_{j+1} = (I - KP)\Psi(v_j) + KP\Psi(v_j)$$

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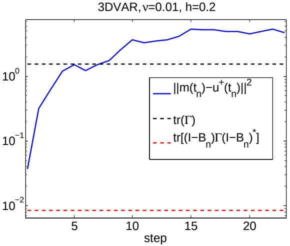
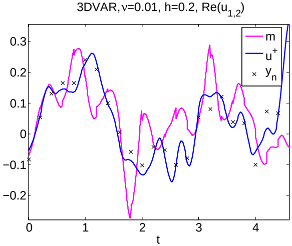
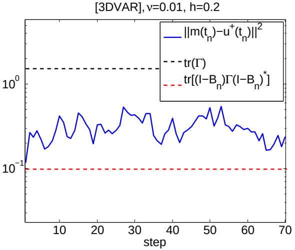
Unstable


3DVAR, ν=0.01, h=0.2
||m(t_n)−u^+(t_n)||^2
tr(Γ)
tr[(I−B_n)Γ(I−B_n)^*]
step


3DVAR, ν=0.01, h=0.2, Re(u_1,2)
m
u^+
y_n
t

Stabilized


[3DVAR], ν=0.01, h=0.2
||m(t_n)−u^+(t_n)||^2
tr(Γ)
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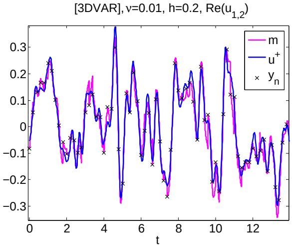
10^0
10^−1
10 20 30 40 50 60 70
step


[3DVAR], ν=0.01, h=0.2, Re(u_1,2)
m
u^+
y_n
0.3 0.2 0.1 0 −0.1 −0.2 −0.3
0 2 4 6 8 10 12
t

Outline

Parameter Scalings

- $\Gamma = \frac{\epsilon^2}{h}\Gamma_0$ and $C = \epsilon^2 r C_0$;
- $y_{j+1} := \left(\frac{z_{j+1} - z_j}{h}\right)$;
- $y_j = \mathrm{P} v_j + \xi_j$;
- $\xi_j \sim \frac{\epsilon}{\sqrt{h}} N(0, \Gamma_0)$.
- $K = rC_0\mathrm{P}^*\left(r\mathrm{P}C_0\mathrm{P}^* + \frac{1}{h}\Gamma_0\right)^{-1}$.

Diffusion Limit

Formal expansion of the 3DVAR filter in h gives:

$$\begin{aligned}\widehat{m}_{j+1} &= \Big(I - hrC_0\mathrm{P}^*(hr\mathrm{P}C_0\mathrm{P}^* + \Gamma_0)^{-1}\mathrm{P}\Big)\Psi(\widehat{m}_j) \\ &\qquad\qquad + hrC_0\mathrm{P}^*(hr\mathrm{P}C_0\mathrm{P}^* + \Gamma_0)^{-1}\Big(\frac{z_{j+1} - z_j}{h}\Big) \\ &\approx \widehat{m}_j - h\big(A\widehat{m} + B(\widehat{m}, \widehat{m}) - f\big) \\ &\qquad\qquad + hrC_0\mathrm{P}^*\Gamma_0^{-1}\Big(\frac{z_{j+1} - z_j}{h} - \mathrm{P}\widehat{m}_j\Big).\end{aligned}$$

The data z evolves according to

$$z_{j+1} = z_j + h\mathrm{P}v_j + \epsilon\sqrt{h}N(0, \Gamma_0).$$

SPDE Limit

Formal diffusion limit. With $\widehat{m}(0) = \widehat{m}_0$ we have

$$\frac{d\widehat{m}}{dt} + A\widehat{m} + B(\widehat{m}, \widehat{m}) + rC_0\mathrm{P}^*\Gamma_0^{-1}\Big(\mathrm{P}\widehat{m} - \frac{dz}{dt}\Big) = f,$$

where, with $z(0) = 0$, the data z solves

$$\frac{dz}{dt} = \mathrm{P}v + \epsilon\sqrt{\Gamma_0}\mathrm{P}\frac{dW}{dt}.$$

Thus

$$\frac{d\widehat{m}}{dt} + A\widehat{m} + B(\widehat{m}, \widehat{m}) + rC_0\mathrm{P}^*\Gamma_0^{-1}\mathrm{P}\Big(\widehat{m} - v\Big) = f + r\epsilon C_0\mathrm{P}^*\Gamma_0^{-1/2}\mathrm{P}\frac{dW}{dt}.$$

Accuracy and Stability Theorem

For all three examples (and others) we have:

Theorem

Assume that:

$$\sup_{t\geq 0}\|v(t)\|^2 = R.$$

Then, for r sufficiently large (depending on R) there is $\gamma, c > 0$ such that

$$\mathbb{E}|\widehat{m}(t) - v(t)|^2 \leq \exp(-\gamma t)\|\widehat{m}(0) - v(0)\|^2 + c\epsilon.$$

Proof of Accuracy/Stability Theorem

$$\frac{dv}{dt} + Av + B(v,v) = f,$$
$$\frac{d\widehat{m}}{dt} + A\widehat{m} + B(\widehat{m},\widehat{m}) + rO_1\mathrm{P}\big(\widehat{m} - v\big) = f + \epsilon O_2 \mathrm{P}\frac{dW}{dt}.$$

Define $e = \widehat{m} - v$ and find:

$$\frac{de}{dt} + Ae + 2B(e,v) + B(e,e) + rO_1\mathrm{P}e = \epsilon O_2\mathrm{P}\frac{dW}{dt}.$$

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Applying the Accuracy/Stability Theorem

For all examples $\exists$, for r sufficiently large, $\gamma > 0$:

$$2|\langle B(a, v), a\rangle| \leq \langle Aa, a\rangle + r\langle \mathrm{O}_1 Pa, a\rangle - \frac{1}{2}\gamma|a|^2$$

Application of the Itô formula (need trace class condition on the noise if we observe everything for NSE) gives

$$\frac{d}{dt}\mathbb{E}|e(t)|^2 \leq -\gamma \cdot \mathbb{E}|e(t)|^2 + c\epsilon\gamma.$$

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Applying Main Theorem

- $C_0 = A^{-2\zeta}$ (Model covariance);
- $\Gamma_0 = A^{-2\beta}$ (Observation covariance);
- define $\alpha = \zeta - \beta \in \mathbb{R}$.
- NSE Nonlinearity estimate:

$$\langle F(a) - F(b), a - b \rangle \leq \frac{1}{2} K \|b\|^2 |a - b|^2 + \frac{\nu}{2} \|a - b\|^2.$$

- interpolation: $\frac{1}{2}\gamma |h|^2 \leq r \langle \mathrm{P} A^{-2\alpha} h, h \rangle + \frac{\nu}{2} \|h\|^2 \quad \forall h \in V.$
- trace class noise: $\mathrm{trace}(A^{-4\alpha - 2\beta}) = \mathrm{c} < \infty.$

Applying Main Theorem

The interpolation inequality reveals restrictions on γ:

$$\frac{1}{2}\gamma|h|^2 \le r\langle A^{-2\alpha}h, h\rangle + \frac{1}{2}\nu\|h\|^2 \qquad \text{for all } h \in \mathrm{P}V$$
$$\frac{1}{2}\gamma \le \frac{r}{|k|^{4\alpha}} + \frac{1}{2}\nu|k|^2 \qquad \text{for all } |k|^2 < \lambda/4\pi^2.$$

$$\gamma|h|^2 \le \langle rA^{-2\alpha}\mathrm{P}h, h\rangle + \nu\|h\|^2 \qquad \text{for all } h \in \mathrm{Q}V$$
$$\gamma \le \nu|k|^2 \qquad \text{for all } |k|^2 \ge \lambda 1/4\pi^2.$$

Need to choose λ, r so that

$$KR < \gamma \le \min\left\{\frac{\nu\lambda}{4\pi^2}, c(\nu^{2\alpha}r)^{1/(2\alpha+1)}\right\}.$$

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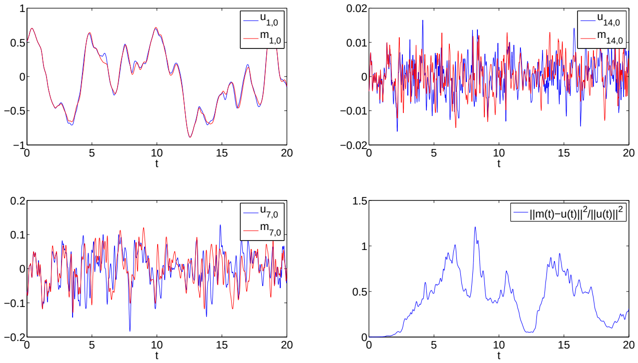
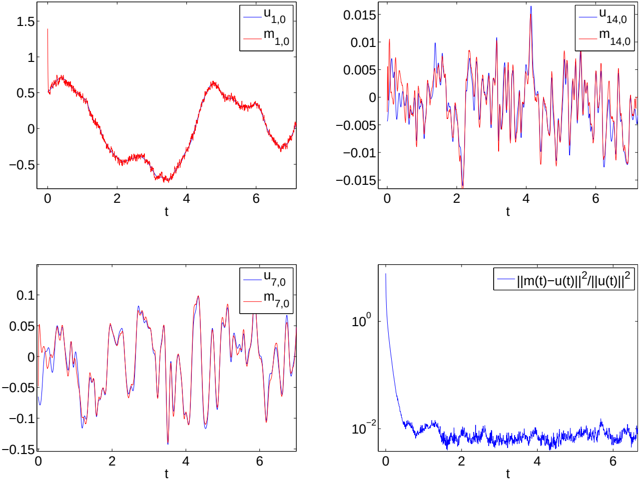
SPDE Unstable

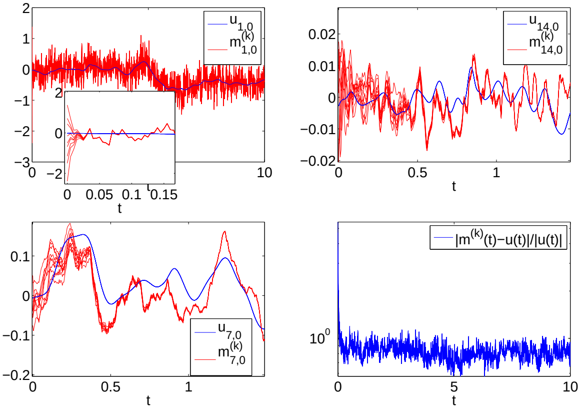


SPDE Stable



1.5
1
0.5
0
−0.5
0
2
4
6
t
$u_{1,0}$
$m_{1,0}$
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
$u_{14,0}$
$m_{14,0}$
0.1
0.05
−0.05
−0.1
−0.15
$u_{7,0}$
$m_{7,0}$
10^0
10^{-2}
$||m(t)-u(t)||^2/||u(t)||^2$

SPDE Inaccurate



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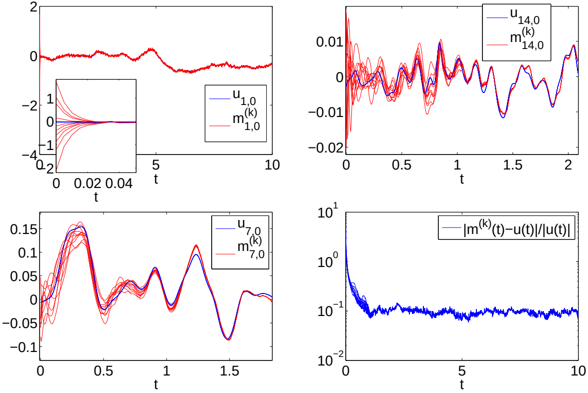
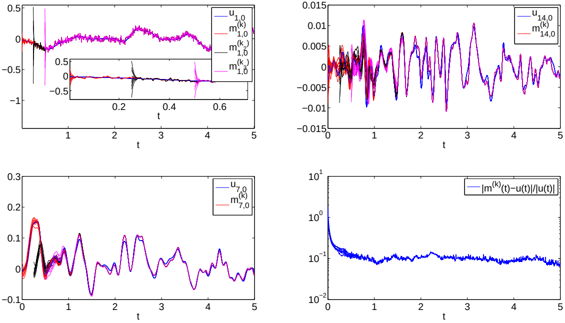


SPDE Pullback



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Conclusions

- Approximate filters are routinely used in geophysical applications.
- They fail to reproduce covariance but can accurately track the mean.
- Observe enough unstable dynamics.
- Model variance inflation: trust the observations.
- SPDE in high frequency in time limit.
- Future work: ExKF, EnKF.

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